December 11, 2012

Limits and Continuity

Grade: «grade»

Subject: «subject»

Date: «date»

1 Answer?

A calculator may not be used on the following questions.

1. Evaluate the limit, if it exists: $\lim_{x \to 2} \frac{x^2 + x - 6}{2 - x}$.

(A) 5
(B) 3
(C) –3
(D) –5
(E) The limit does not exist.

$\lim_{x \to 2} \frac{(x+3)(x-2)}{-(x-2)}$

$= \frac{(2+3)}{-1} = -5$

2 Answer?

2. Evaluate the limit, if it exists: $\lim_{x\to 9}\dfrac{\sqrt{x-5}-2}{x-9}$.

(A) $\dfrac{1}{4}$

(B) $-\dfrac{1}{4}$

(C) 1

(D) 0

(E) The limit does not exist.

$\dfrac{(\sqrt{x-5}+2)}{(\sqrt{x-5}+2)}$

$\dfrac{x-5-4}{(x-9)(\sqrt{x-5}+2)} = \dfrac{x-9}{x-9\,(\sqrt{x-5}+2)}$

$\lim_{x\to 9} \dfrac{1}{\sqrt{x-5}+2}$

$= \dfrac{1}{4}$

3 Answer?

3. Evaluate the limit, if it exists: $\lim_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x - 2}$.

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) 1

(D) –1

(E) The limit does not exist.

$$\lim_{x \to 2} \frac{\frac{2-x}{2x}}{x-2}$$

$$\lim_{x \to 2} \frac{2-x}{2x} \cdot \frac{1}{x-2}$$

$$\lim_{x \to 2} \frac{-(x-2)}{2x} \cdot \frac{1}{x-2}$$

$$= -\frac{1}{4}$$

4 Answer?

domain $\tan x$: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

4. Evaluate the limit, if it exists: $\lim_{x \to 1} \frac{\tan^{-1} x}{\sin^{-1} x + 1}$.

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

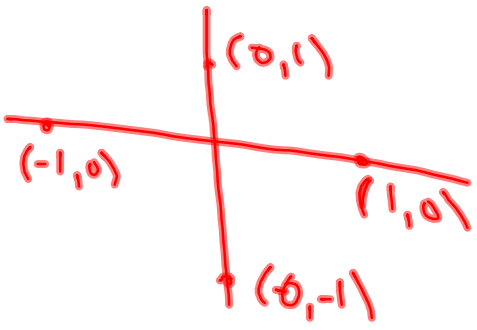
(D) $\frac{\pi}{2}$

(E) $\frac{\pi}{2\pi + 4}$

$\frac{\frac{\pi}{4}}{\frac{\pi}{2} + 1}$

$\frac{\frac{\pi}{4}}{\frac{\pi + 2}{2}}$ $\frac{\pi}{2} \cdot \frac{2}{(\pi + 2)}$

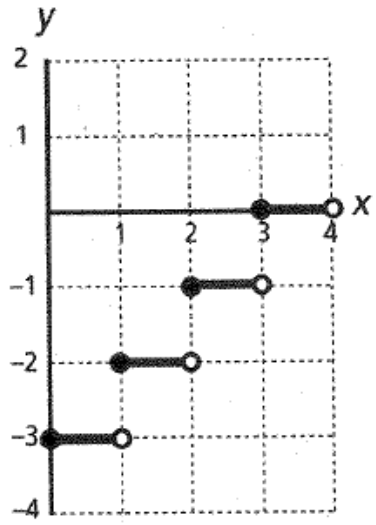
$= \frac{\pi}{2\pi + 4}$



5 Answer?

Estimate the limit, if it exists: $\lim_{x \to 3} f(x)$ where $f(x)$ represented by the given graph:

(A) 0
(B) –1
(C) 3
(D) 1
(E) The limit does not exist.



$\lim_{x \to 3^-} f(x) = -1$

$\lim_{x \to 3^+} f(x) = 0$

lim does not exist!

6 Answer?

6. Given the function

$$f(x) = \begin{cases} \sin 2x, & x \le \pi \\ 2x + k, & x > \pi \end{cases}$$

what value of k will make this piecewise function continuous?

(A) -2π
(B) $-\pi$
(C) 0
(D) π
(E) 2π

$\lim_{x \to \pi^-} f(x)$ $\lim_{x \to \pi^+} f(x)$

$\sin 2\pi = 2\pi + k$

$0 = 2\pi + k$

$k = -2\pi$

7 Answer?

7. Find the limit, if it exists: $\lim_{x \to 0} x\left(e^x + \frac{1}{x}\right)$.

(A) 0
(B) 1
(C) 2
(D) The limit does not exist.
(E) None of these

$\lim_{x \to 0} xe^x + 1$

$= 0 + 1$

8 Answer?

8. Identify the vertical asymptotes for $f(x) = \dfrac{x^2 + 3x - 4}{x^2 + x - 2}$.

(A) $x = -2, x = 1$
(B) $x = -2$
(C) $x = 1$
(D) $y = -2, y = 1$
(E) $y = -2$

$$= \frac{(x+4)(x-1)}{(x+2)(x-1)}$$

hole at $x = 1$

vertical asymptote, $x = -2$

9 Answer?

9. If $p(x)$ is a continuous function on the closed interval [1, 3], with $p(1) \le K \le p(3)$ and c is in the closed interval [1, 3], then which of the following statements must be true?

(A) $p(c) = \dfrac{p(3) + p(1)}{2}$

(B) $p(c) = \dfrac{p(3) - p(1)}{2}$

(C) There is at least one value c such that $p(c) = K$.

(D) There is only one value c such that $p(c) = K$.

(E) $c = 2$

Intermediate Value Theorem

10 Answer?

10. How many vertical asymptotes exist for the function $f(x) = \dfrac{1}{2\sin^2 x - \sin x - 1}$ in the open interval $0 < x < 2\pi$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$f(x) = \dfrac{1}{(2\sin x + 1)(\sin x - 1)}$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin x = 0$ $x = \frac{\pi}{2}$

$\sin x = 1$

$2x^2 - x - 1$

$(2x + 1)(x - 1)$

11 Answer?

x	1	2	3	4
$f(x)$	4	2	3	1
$g(x)$	2	3	1	4

11. Selected values for continuous functions $f(x)$ and $g(x)$ are given in the table above. $\lim_{x \to 3} \frac{f(g(x))}{g(f(x))} =$

(A) $\frac{1}{4}$
(B) $\frac{1}{3}$
(C) 1
(D) 3
(E) 4

$$\frac{f(1)}{g(3)} = \frac{4}{1}$$

12 Answer?

12. $\lim_{x \to \infty} \frac{\sin x}{e^x + \cos x} =$

(A) -1

(B) 0

(C) $\frac{1}{e}$

(D) 1

(E) The limit does not exist.

e^x gets bigger as $x \to \infty$

13 Answer?

13. For what value of k is the function $f(x) = \begin{cases} \dfrac{2x^2+5x-3}{x^2-9}, & x \neq -3 \\ k, & x = -3 \end{cases}$ continuous at $x = -3$?

(A) $-\frac{7}{6}$

(B) $-\frac{5}{6}$

(C) 0

(D) $\frac{5}{6}$

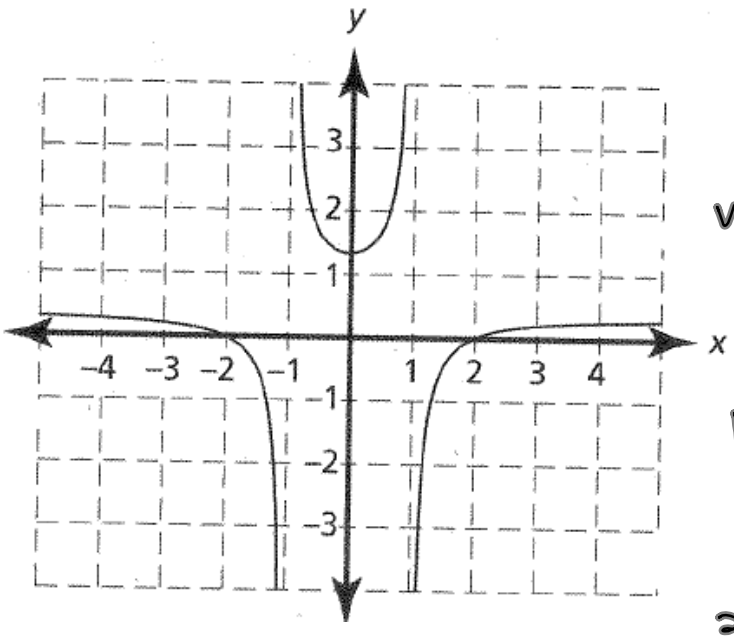
(E) $\frac{7}{6}$

$$\frac{(2x-1)(x+3)}{(x+3)(x-3)}$$

$$\frac{2x-1}{x-3} = ? \text{ when } x=-3$$

$$= \frac{7}{6}$$

14 Answer?



vertical asymptotes at $x=-1$, $x=1$

bx^2-3 must $=0$ at $x=-1$ and $x=1$

$3x^2-3$

$3(x^2-1) = 3(x+1)(x-1)$

14. The function $g(x)$ is shown in the graph above and is of the form $g(x)=\frac{x^2+a}{bx^2-3}$. Which of the following could be the values of the constants a and b?

(A) $a=-2,\ b=-1$
(B) $a=-2,\ b=-3$
(C) $a=-4,\ b=3$
(D) $a=-4,\ b=-3$
(E) $a=4,\ b=3$

zeroes: $x=2$, $x=-2$

x^2+a must $=0$ at 2 and -2

x^2-4 $(x+2)(x-2)$

$a=-4$

15 Answer?

15. Which of the following statements is true?

(A) $\lim_{x \to 3} \log_3 x = 2$ $\log_3 3 = 1$

(B) $\lim_{x \to 0^+} \log_3 x$ does not exist.

(C) $\lim_{x \to -\infty} e^x$ does not exist. $= 0$

(D) $\lim_{x \to -\frac{\pi}{2}} \csc x = 1$ $= -1$

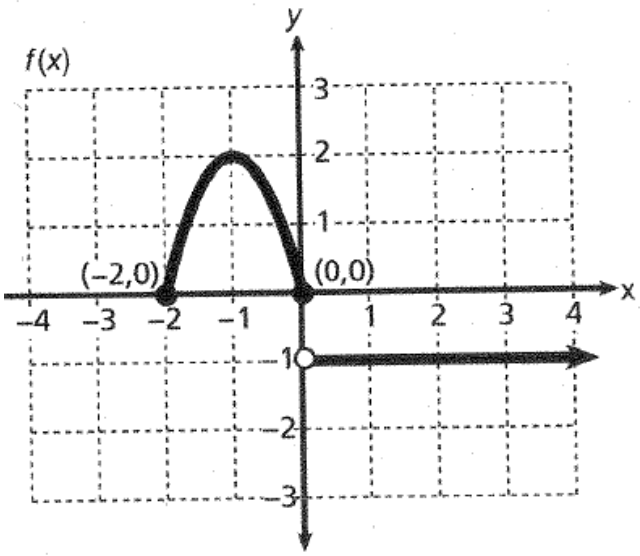
(E) $\lim_{x \to 1} e^{x-1} = 0$ $= 1$

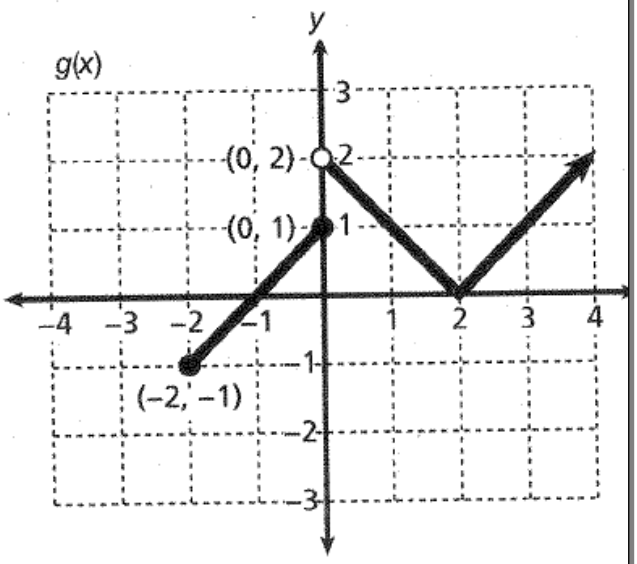
$\frac{1}{\sin x}$

FREE-RESPONSE QUESTION

A calculator may not be used for this question.

1. Use the graphs of $f(x)$ and $g(x)$ given below to answer the following questions:





a. Is $f[g(x)]$ continuous at $x = 0$? Explain why or why not.

b. Is $g[f(x)]$ continuous at $x = 0$? Explain why or why not.

c. What is $\lim_{x \to \infty} f[g(x)]$? Explain your reasoning.

d. If $h(x) = \begin{cases} f(x) + g(x), & -2 \le x \le 0 \\ k + g(x)f(x), & x > 0 \end{cases}$, what is k so that $h(x)$ is continuous at $x = 0$?

December 11, 2012

Derivative

Grade: «grade»

Subject: «subject»

Date: «date»

Calculators may not be used on this part of the examination.

1 Answer?

1. What does the limit statement $\lim_{x \to 1} \frac{\ln(x+1) - \ln 2}{x-1}$ represent?

(A) 0

(B) $\frac{d}{dx}[\ln(x+1)]$

(C) $f'(1)$, if $f(x) = \ln(x+1)$

(D) 1

(E) The limit does not exist.

2 Answer?

2. Find the derivative of the function $y = \frac{4}{x^3}$.

(A) $-4x^2$

(B) $-\frac{12}{x^2}$

(C) $\frac{12}{x^2}$

(D) $\frac{12}{x^4}$

(E) $-\frac{12}{x^4}$

3 Answer?

3. Find $\frac{dy}{dx}$ if $3xy = 4x + y^2$.

(A) $\frac{4-3y}{2y-3x}$

(B) $\frac{3x-4}{2x}$

(C) $\frac{3y-x}{2}$

(D) $\frac{3y-4}{2y-3x}$

(E) $\frac{4+3y}{2y+3x}$

4 Answer?

4. Find $\frac{dy}{dx}$ for $e^{x+y} = y$.

(A) $\frac{e^{x+y}}{\left(1-e^{x+y}\right)}$

(B) $\frac{e^{x+y}}{\left(1+e^{x+y}\right)}$

(C) $\frac{e^{x+y}}{\left(e^{x+y}-1\right)}$

(D) e^{x+y}

(E) $2e^{x+y}$

5 Answer?

5. If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(7)}$ is the same as

(A) y

(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) None of these

6 Answer?

6. Find the second derivative of $f(x)$ if $f(x) = (2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $12(2x+3)^2$

(D) $24(2x+3)^2$

(E) $48(2x+3)^2$

7 Answer?

A calculator may be used for any of the following multiple-choice questions.

7. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.

(A) $8\sin(3x)$
(B) $24\sin(3x)$
(C) $8\sin(3x)\cos(3x)$
(D) $12\sin(3x)\cos(3x)$
(E) $24\sin(3x)\cos(3x)$

8 Answer?

If $\ln y = (\ln x)^2 + 2$, find $\frac{dy}{dx}$ in terms of x and y.

(A) $y\left[2\ln(x)+\frac{1}{x}\right]$

(B) $y\left[\left(\frac{2}{x}\right)\ln(x)\right]$

(C) $\left(\frac{2}{x}\right)\ln(x)$

(D) $\frac{2(\ln x)}{x}+2$

(E) $y\left[\frac{2(\ln x)}{x}+2\right]$

9 Answer?

If $f(2) = -3$, $f'(2) = \frac{3}{4}$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

(A) $y - 2 = \frac{-3}{4}(x + 3)$

(B) $y + 2 = \frac{-3}{4}(x - 3)$

(C) $y - 2 = \frac{-4}{3}(x + 3)$

(D) $y + 2 = \frac{4}{3}(x - 3)$

(E) $y - 2 = \frac{4}{3}(x + 3)$

10 Answer?

For what positive value of x does the tangent line to the curve $y = \ln(1 - x)$ intersect the y-axis at the point (0, 2)?

(A) 0.382
(B) 0.547
(C) 0.667
(D) 0.722
(E) 0.778

11 Answer?

Calculators may not be used for this part of the examination.

For what values of a and c is the piecewise function

$$f(x)=\begin{cases} ax^2+\sin x, & x \le \pi \\ 2x-c, & x>\pi \end{cases}$$ differentiable?

(A) $a=\dfrac{3\pi}{2}$ and $c=\dfrac{\pi}{2}$

(B) $a=\dfrac{3}{2\pi}$ and $c=\dfrac{7\pi}{2}$

(C) $a=\dfrac{3}{2\pi}$ and $c=-\dfrac{\pi}{2}$

(D) $a=\dfrac{3}{2\pi}$ and $c=\dfrac{\pi}{2}$

(E) $a=\dfrac{3\pi}{2}$ and $c=\dfrac{2}{\pi}$

12 Answer?

If $y = \tan^{-1}(x^2 + 3x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+(x^2+3x)^2}$

(B) $\frac{1}{x^2+3x+1}$

(C) $\frac{2x+3}{1+(x^2+3x)^2}$

(D) $\frac{2x+3}{(x^2+3x)^2}$

(E) $\frac{x^2+3x}{1+(x^2+3x)^2}$

13 Answer?

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	1	−2	4
2	5	3	1	−4
3	2	1	−2	1
4	4	−3	2	−1

Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are given in the table above. If $p(x) = x \cdot f(x) - g(3x-2)$, then $p'(2) =$

(A) 11
(B) 10
(C) 8
(D) 6
(E) 4